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How to distribute the ERDF funds through a combination of egalitarian allocations: the CELmin

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Abstract

As Solís-Baltodano et al. (2021) figure out, almost a third of the total European Union budget– has been set aside for the Cohesion Policy during the 2014-2020 period. The distribution of this budget is made through three main structural and investment funds, trying to provoke the convergence in the level of development of EU countries. Specifically, the authors, by analysing this situation as a claims problem (O'Neill, 1982), find out the claims solution that performs better than the others by reducing inequality promoting convergence to a greater degree (the constrained equal losses rule). Nonetheless, when using this egalitarian division of losses, regions may receive no amount of funds. This paper defines a new way to distribute the limited resources of the European Regional Development Fund (ERDF). We propose a compromise between the egalitarian approaches, i.e., we combine the egalitarian division of the funds with an egalitarian division of the losses (what regions do not get). In doing so, our proposal combines the constrained equal losses solution with the ensuring of a minimum amount to each region (sustainable bound). Finally, we provide an axiomatic analysis of the new solution and we apply it to the ERDF problem.

Keywords: European Regional Development Fund; Conflicting claims problems; Egalitarian distribution; Constrained equal losses

1. Introduction

European regional development fund (ERDF) is part of the European structural and investment fund, which has been designed under the EU cohesion policy. With the aim of reducing inequalities in the level of development among regions throughout EU and compensate the backwardness of less developed regions, ERDF is invested to support the medium and small size business, improve the health system, develop the digital infrastructure, enforce non-polluted transportation and reducing the greenhouse gas emission to achieve the target of carbon neutral by 2050.

European Commission together with member states are responsible for allocating ERDF budget to regions. To allocate ERDF, each member state is classified to three regions according to the GDP per capita: less developed, transition and more developed regions; and ERDF is distributed to cover the need of the regions according to so called Berlin method. However, if we consider the ERDF budget as the endowment to be allocated, and the claims consist of the amounts required to develop some projects (mainly in infrastructures: airports, universities, hospitals, etc.) that regions could not afford individually, it is noteworthy that the available budget is not enough to satisfy all the claims that the regions have on it, thus, we have a claims problems (O'Neill, 1982).

Within this context, the current approach complements Fragnelli and Kiryluk-Dryjska (2019) and Solís-Baltodano et al. (2021), by defining a new way of distributing ERDF. As Fragnelli and Kiryluk-Dryjska (2019) mentions "this approach has the great advantage that solutions may be obtained with a fast computation." Particularly, Solís-Baltodano et al. (2021) identify the agents (*the EU NUTS level 2 regions*) and the endowment (*the ERDF budget to be allocated*) and use four claims solutions: the proportional rule, the constrained equal awards rule, the constrained equal losses rule, and the $\alpha_{\min}$-Egalitarian rule. Among the analysed rules, the one that performs best (promoting convergence) is the one that proposes the most unequal (*per capita*) distribution of the ERDF budget: the constrained equal losses rule.

It is noteworthy that there are other related economic and social problems where the claims approach is implemented: in the education sector Pulido et al. (2002) use this approach for obtaining an efficient allocation of the university funds; in the fishing sector, it is a useful tool for searching possible solutions to address fish shortages, by proposing fishing quotas among a number of agents within an established perimeter (Iñarra and Prellezo, 2008;

Iñarra and Skonhoft, 2008; Kampas, 2015); Kiryluk-Dryjska (2014, 2018) that propose a formal framework for rural development budget allocation by using fair division techniques; or, in the negotiations on CO2 emissions, a relevant issue nowadays, Giménez-Gómez et al. (2016) and Duro et al. (2020) propose an appealing distribution by analyzing this situation as a conflicting claims problem.[1]

To solve claims problems, we have several division rules which propose a unique way to divide the endowment among agents. As aforementioned, Solís-Baltodano et al. (2021) study the allocation of ERDF as a claims problems by investigating different division rules. The authors show that the constrained equal losses (CEL) solution, an egalitarian rule which divides the difference between the aggregate claims and the endowment (the part that cannot be honoured, i.e., the losses) equally to the agents is the best proposal to achieve the target of convergence in EU. The CEL gives priority to agents with larger claims, that is, the less developed regions.

Nonetheless, the CEL assigns no funds to some regions, hence, it is usually not applied in real situations (no regions will accept not receiving any amount).[2] Therefore, it seems clear that, in any real situation, smaller claimants should be protected.

Following Giménez-Gómez and Peris (2014), we propose to guarantee a minimal amount for all the claimants, and, then, divide the remainder by the CEL solution. This minimal amount is based on the sustainability and preeminence concepts (Herrero and Villar, 1998, Herrero and Villar, 2002). On the one hand, if region i's claim is as small as when we truncate the claims of other regions to agent i's claim, we do not have claims problem anymore, then, this claim is sustainable. Sustainability states that these types of claims should be completely honoured. On the other hand, we should take into account the excess of claim, i.e., the losses. Preeminence, which is the dual concept of sustainability, establishes that if a claim is removed from the problem, and we still have claims problem, then this, so called, residual claim should not be satisfied. Hence, preeminence, which is satisfied by the CEL solution, gives priority to larger claims, the claims which can change

[1] We refer the reader to well-known classic studies in this context , Young (1987), Auman and Maschler (1985), Thomson (2003).

[2] See, for instance, Solís-Baltodano et al. (2021), where regions with smaller claims receive nothing from the ERDF, or Duro et al. (2020), where small countries receive no CO2 emission permission.

the situation of the claims problem.

Our proposed solution, called CELmin, keeps a balance between sustainability and preeminence. CELmin compromises the egalitarian distribution of the endowment with CEL solution. The rule proposes that: if the smallest claim is sustainable, CELmin assigns a minimal guarantee equal to the smallest claim to all agents (or the egalitarian distribution of the endowment), and revises down the claims and the endowment to implement CEL and distributing the remaining. If not, the rule proposes an equal division of the state.

The rest of the paper is organized as follows. In Section 2 we formally present the notion of claims problems and some of the main solutions in the literature. In Section 3 we define our new solution, the CELmin. In Section 4 we make an axiomatic analysis of the proposed solution. In Section 5 we apply the previous analyses to the ERDF problem and Section 6 analyses and compares the proposed allocations from the point of view of convergence. Some final comments in Section 7 conclude the paper.

2. Preliminaries: claims problems

The agents are defined as a set of $N = \{1, 2, ..., n\}$. Each agent is identified by her *claim*, c_i,$i \in N$, on the *endowment* E. A **claims problem** occurs when the endowment is not sufficient to cover all the claims, that means $\sum_{i=1}^{n} c_i > E$. Without loss of generality, we order agents according to their claims: $c_1 \leq c_2 \leq \cdots \leq c_n$. The pair (E, c) represents the claims problem and $\mathcal{B}$ is the set of all claims problems. A *claim rule* (**solution**) is a single value function $\varphi : \mathcal{B} \to \mathbb{R}^n_+$ such that, for each $i \in N$, $0 \leq \varphi_i(E, c) \leq c_i$, (**non-negativity** and **claim-boundedness**) and $\sum_{i=1}^{n} \varphi_i(E, c) = E$, (**efficiency**).

Next we define four well-known classic solutions of claims problems (see Thomson (2003)).

Definition 1. ***Proportional*** *(P) divides the endowment proportionally according to agents' claims.*

For each $(E, c) \in \mathcal{B}$ *and each* $i \in N$, $P_i(E, c) = \lambda c_i$, *where* $\lambda = \dfrac{E}{\sum_{i \in N} c_i}$.

Definition 2. ***Equal Awards division*** *(EA) assigns the endowment equally* *among all members. It is easy to see that in some situations with the equal distribution may occur that an agent receives more than her claim.*

For each $(E,c) \in \mathcal{B}$ *and each* $i \in N$, $EA_i(E,c) = \frac{E}{n}$.

Definition 3. ***Constrained Equal Awards*** *(CEA) assigns the endowment equally by imposing a constraint on the allocation, such that no agent receives more than her claim.*

For each $(E,c) \in \mathcal{B}$ *and each* $i \in N$, $CEA_i(E,c) \equiv \min\{c_i, \mu\}$, *where* μ *is chosen so that* $\sum_{i \in N} \min\{c_i, \mu\} = E$.

Definition 4. ***Constrained Equal Losses*** *(CEL) allocates the loss which is the difference between aggregate claims and the endowment. This measure is divided equally, such that no agent receives negative amount.*

For each $(E,c) \in \mathcal{B}$ *and each* $i \in N$, $CEL_i(E,c) \equiv \max\{0, c_i - \mu\}$, *where* μ *is such that* $\sum_{i \in N} \max\{0, c_i - \mu\} = E$.

In addition, we mention $\alpha_{\min}$-Egalitarian rule (Giménez-Gómez and Peris (2014)), which is a compromising ofthe *Equal Awards division* and the *Proportional.*

Definition 5. $\alpha_{\min}$***-Egalitarian*** *(*$\alpha_{\min}$*) guaranties a minimal right equals the smallest claim to all agents, if the endowment is sufficient and distributes proportionally the remaining endowment to agents' revised claims. If the endowment is not enough, it is divided equally.*

For each $(E,c) \in \mathcal{B}$ *and each* $i \in N$, *if* $c_1 > \frac{E}{n}$ *then* $\alpha_{min_i}(E,c) = \frac{E}{n}$ *and if* $c_1 < \frac{E}{n}$ *then* $\alpha_{min_i}(E,c) = c_1 + P(E - nc_1, c - c_1)$.

3. The CELmin solution

By combining the *Constrained Equal Loses* and the *Egalitarian division* we define a solution that, if possible, assigns the minimal positive claim to any agent and distributes the remaining estate $E' = E - nc_1$ by implementing the CEL rule among the agents with respect to the remaining claims $c_i' = c_i - c_1$. If the estate is not enough to assign the minimal claim to any agent, then we assign the equal division of the endowment to all agents.

Definition 6. *For each* $(E, c) \in \mathcal{B}$ *with* $c_i > 0$ *and each* $i \in N$,

$$CELmin(E,c) = \begin{cases} (E/n)\boldsymbol{1} & \text{if } c_1 \geq E/n \\ \boldsymbol{c}^1 + CEL(E - nc_1, c - \boldsymbol{c}^1) & \text{otherwise} \end{cases}$$

where $\boldsymbol{c}^1 = (c_1, \ldots, c_1)_{1xn}$ *and* $\boldsymbol{1} = (1, \ldots, 1)_{1xn}$

In case that some claims are equal to zero, $c_1 = c_2 = \ldots c_k = 0$, $c_j > 0$, *for each* $j > k$, *we extend the solution in a consistent manner:*

$$CELmin(E,c) = (\boldsymbol{0}, CELmin(E, \bar{c}))$$

where $\boldsymbol{0} = (0, \ldots, 0)_{1xk}$ *and* $\bar{c} = (c_{k+1}, \ldots, c_n)$.

The following example shows how the rule proceeds.

Example 1. *Consider* $(E, c) = (2000; (500, 2000, 2400)$. $CELmin(E,c) = (500, 500, 500) + CEL(500, (0, 1500, 1900) = (500, 500, 500) + (0, 0, 500) = (500, 500, 1000)$. *Notice that with the CEL we have* $CEL(E, c) = (0, 800, 1200)$, *agent one receives nothing and with the CELmin every one receives at least a minimal amount of 500.*

Compared with Proportional rule which allocates $P(E, c) = (204.08, 816.33, 979.59)$, *in this example larger claimants receive larger amount by applying CELmin (in section 6 we see that this is not always the case) . Although,CELmin and* α_{min} *assign equal minimal right, but the allocation of* $\alpha_{min}(E, c) = (500, 750.59, 779.41)$ *shows CELmin protects larger claimants more than* α_{min}. *Constrained equal awards is the rule which allocated smallest share to larger claimants,* $CEA(E, c) = (500, 750, 750)$.

Similarly, Hougaard et al. (2013a) and Alcalde and Peris (2022) provide insights also about the combination of equal sharing losses and awards. Specifically, Hougaard et al. (2013a) ensure each claimant an endogenous minimal amount that depends on the claims and the endowment, called baseline, b. For each $(N, b, c, E) \in \mathcal{E}$, let $t_i(b, c) = \min(b_i, c_i)$ for each $i \in N$ and $t(b, c) = \{t_i(b, c)\}_{i \in N}$ denotes the truncated baseline-claim vector and $T = \sum\limits_{i \in N} t_i(b, c)$. In this context, the authors define a family of rules, S^b, throughout a composition operator (Hougaard et al. (2012) and Hougaard et al. (2013b)) as $S^b(N, b, c, E) = S(N, t(b, c), E)$, if $E \leq T(b, c)$, or $S^b(N, b, c, E) = t(b, c) + S(N, c - t(b, c), E - T(b, c))$, if $E \geq T(b, c)$.

It is noteworthy that if we define the baseline as the smallest claimant and take CEL as the starting rule then, the CELmin is retrieved,

$$S_i^b(N, b, c, E) = c_1 + CEL_i(N, c - c_1, E - nc_1)$$

4. Axiomatic analysis

In this section we analyze the previous rule from an axiomatic point of view and we compare it with the Constrained Equal Loses which is the rule that is most related to it. At the end of the section there is a table with a summary of the axiomatic comparative of the axioms siatisfied by these two rules. Next, we define the axioms with the aim to study whether the CELmin satisfies them or not.

Order preservation (Auman and Maschler (1985)) considers that the order of the claims must be respected. If agent i's claim is at least as large as agent j's claim, the awards and losses allocated to agent i must be at least as much as the ones allocated to agent j.

> **Order preservation:** for each $(E, c) \in \mathcal{B}$, and each $i, j \in N$, such that $c_i \geq c_j$, then $\varphi_i(E, c) \geq \varphi_j(E, c)$, and $c_i - \varphi_i(E, c) \geq c_j - \varphi_j(E, c)$.

Resource monotonicity (Curiel et al. (1987)) says that if the endowment increases, all agents should receive at least the measure that was allocated to them before increasing.

> **Resource monotonicity:** for each $(E, c) \in \mathcal{B}$ and each $E' \in \mathbb{R}_+$ such that $C > E' > E$, then $\varphi_i(E', c) \geq \varphi_i(E, c)$, for each $i \in N$.

Super-modularity (Dagan et al., 1997) requires that if the endowment increases, given two agents, the one with greater claim should receive greater portion of the increment than the other.

> **Super-modularity:** for each $(E, c) \in \mathcal{B}$, all $E' \in \mathbb{R}_+$ and each $i, j \in N$ such that $C > E' > E$ and $c_i \geq c_j$, then $\varphi_i(E', c) - \varphi_i(E, c) \geq \varphi_j(E', c) - \varphi_j(E, c)$.

Order preservation under claims variations(Thomson (2019) demands that if the claim of one agent decreases, given two other agents, the one with the greater claim receive more than the other.

Order preservation under claims variations: for each $k \in N$, each pair (E,c) and $(E,c') \in \mathcal{B}$, with $c' = (c'_k, c_{-k})$ and $c'_k < c_k$ and each pair i and $j \in N \setminus k$ with $c_i \leq c_j$, $\varphi_i(E,c') - \varphi_i(E,c) \leq \varphi_j(E,c') - \varphi_j(E,c)$[3].

Composition down requires that, if after distributing the endowment, the measure of endowment decreases, two options are available: first, cancel the initial allocation and apply the rule for the revised endowment. Second, consider the agents' initial awards as their claims and apply the rule to allocate the revised endowment in this situation. Both ways should lead the same award vector.

Composition down: for each $(E,c) \in B$, each $i \in N$, and each $0 \leq E' \leq E$, $\varphi_i(E',c) = \varphi_i(E', \varphi(E,c))$.

Limited consistency states that adding an agent with a zero claim does not affect the award of other agents [4].

Limited consistency: for each $(E,c) \in \mathcal{B}$ and each $i \in N$, $\varphi_i(E,c) = \varphi_i(E,(0,c_1,...,c_n))$.

In the next proposition we show that the CELmin solution satisfies all the axioms mentioned above.

Proposition 1. *The CELmin solution satisfies Order preservation, Resource monotonicity, Super modularity, Order preservation under claims variation, composition down and Limited consistency.*

Proof. For each $(E,c) \in \mathcal{B}$, if $c_1 \geq \frac{E}{n}$, then CELmin equals EA rule. Otherwise each agent receives $CEL_{\min}(E,c) = c_1 + CEL(E - nc_1, c - c_1)$, then the rule satisfies Order preservation.

Regarding the Resource monotonicity, the only considerable case is when $c_1 \geq \frac{E}{n}$ and $c_1 < \frac{E'}{n}$, then $CELmin(E,c) = \frac{E}{n}$ and $CELmin(E',c) = c_1 +$

[3] Notice that (c'_k, c_{-k}) is the claims vector obtained from c by replacing c_k by c'_k.

[4] Clearly if $(E,(c_1,\ldots,c_n))$ is a claims problems with n agents, then $(E,(0,c_1,\ldots,c_n))$is a claims problems with n+1 agents.

$CEL(E' - nc_1, c - c_1)$ which shows that the axiom is satisfied. Similarly we can prove that the rule satisfies Super modularity.

To prove that the rule satisfies Composition down, we confine to the following condition: If $c_1 < \frac{E'}{n} \leq \frac{E}{n}$, then $CELmin(E, c) = c_1 + CEL(E - nc_1, c - c_1)$ and $CEL_{\min}(E', c) = c_1 + CEL(E' - nc_1, c - c_1)$. Based on the definition of Composition down, we have $c_1 + CEL(E' - nc_1, c - c_1) = c_1 + CEL(E' - nc_1, +CEL(E - nc_1, c - c_1))$.

To show Order preservation under claims variations is fulfilled by the rule, we must distinguish three cases:

1. If $c_1 \geq c'_1 \geq \frac{E}{n}$, then, $CELmin(E, c) = CELmin(E, c') = \frac{E}{n}$.
2. If $c_1 \geq \frac{E}{n} > c'_1$, then, $CELmin(E, c) = (\frac{E}{n})$ and $CELmin(E, c') = c_1 + CEL(E - nc'_1, c - c'_1)$.
   According to the definition of Order preservation under claims variations and the fact that the CEL satisfies the property we have: $c_1 + CEL_i(E - nc'_1, c - c'_1) - \frac{E}{n} \leq c_1 + CEL_j(E - nc'_1, c - c'_1) - \frac{E}{n}$.
3. If $c_1 < \frac{E}{n}$, then $CELmin(E, c) = c_1 + CEL(E - nc_1, c - c_1)$.
   (a) If $k = 1$, $c' = (c'_1, c_{-1}) = (c'_1, c_2, c_3, \ldots)$, $c'_1 < c_1$. Then, $c'_1 + CEL_i(E - nc'_1, c' - c'_1) - c_1 - CEL_i(E - nc_1, c - c_1) \leq c'_1 + CEL_j(E - nc'_1, c' - c'_1) - c_1 - CEL_j(E - nc_1, c - c_1)$.
   (b) If $k \neq 1$, $c_1 = c'_1 < \frac{E}{n}$ and $c' = (c'_k, c_{-k}) = (c_1, \ldots, c'_k, \ldots)$. Then, $c_1 + CEL_i(E - nc_1, c' - c_1) - c_1 - CEL_i(E - nc_1, c - c_1) \leq c_1 + CEL_j(E - nc_1, c' - c_1) - c_1 - CEL_j(E - nc_1, c - c_1)$, again since the CEL satisfies the property it is satisfied.

   Finally, we prove that CELmin satisfies Limited consistency. We must distinguish two cases:
   (a) If $c_1 \geq \frac{E}{n}$, then $c_1 > 0$ and we have: $CELmin(E, (0, c_1, \ldots, c_n)) = (0, \frac{E}{n}, \ldots, \frac{E}{n}) = CELmin(E, (c_1, \ldots, c_n))$.
   (b) If $c_1 < \frac{E}{n}$, then $CELmin(E, (0, c_1, \ldots, c_n)) = (0, c_1, \ldots, c_1) + CEL(E - nc_1, (0, c - c_1)) = (0, c_1, \ldots, c_1) + (0, CEL(E - nc_1, c - c_1) = CELmin(E, (c_1, \ldots, c_n))$

■

Albeit CELmin and CEL have shown similar behavior in axiomatic analysis fo far, Composition up and Invariance under claims truncation are two axioms which are satisfied by CEL but not fullfiled by CELmin.

Composition up demonstrates the opposite situation of Composition down in which after distributing the endowment, re-evaluation shows the endowment increase. Again, two ways are available: First, cancel the initial distribution and apply the rule for revised endowment. Second, the claims of agents are revised down by their initial gains. The rule divides the increment part of the endowment to revised claims. The result of both ways should coincide.

> **Composition up:** for each $(E', c) \in B$, each $i \in N$, and each $0 \leq E \leq E'$, $\varphi_i(E', c) = \varphi_i(E, c) + \varphi_i(E' - E, c - \varphi(E, c))$.

The following example shows that the CELmin does not satisfy Composition up.

Example 2. *Consider* $(E, c) = (30, (10, 20, 30))$. *Then,* $CELmin(E, c) = (10, 10, 10)$. *If the endowment increases to* $E' = 50$, *according to the definition of Composition up, the below equation should be obtained:*

$$CELmin(50, (10, 20, 30) = CELmin(30, (10, 20, 30) + CELmin(20(0, 10, 20).$$

But, $CELmin(E', c) = (10, 15, 25)$ *and* $CELmin(E, c) + CELmin(20, (0, 10, 20) = (10, 20, 20)$ *which does not coincide, therefore Composition Up is not satisfied.*

Invariance under claims truncation requires the part of the claim of the agent i that exceed the endowment should be ignored. Indeed, agent i cannot ask more than the available resource.

> **Invariance under claims truncation:** for each $(E, c) \in B$, $\varphi(E, c) = \varphi(E, \min\{c_i, E\})$.

To show that the CELmin does not fulfil Invariance under claims truncation and all the following axioms we refer to *Example 1.* Next we define the

axioms that the CELmin does not satify and are not met by CEL as well.

Self duality: requires for the solution to recommend the same allocation when dividing gains and losses, where losses are defined as the difference among the sum of the claims and the state.

Self duality for each $(E, c) \in B$, and each $i \in N$, $\varphi_i(E, c) = c_i - \varphi_i(L, c)$.

Midpoint property ensures to each agent half of her claim when the estate is equal to half of the aggregate claim.

Midpoint property: for each $(E, c) \in B$, and each $i \in N$, if $E = C/2$, then $\varphi_i(E, c) = c_i/2$.

Reasonable lower bounds on awards: ensures that each individual receives at least the minimum of her claim and the endowment divided by the number of individuals.

Reasonable lower bounds on awards for each $(E, c) \in B$, and each $i \in N$, if $E = C/2$, then $\varphi_i(E, c) \geq \frac{\min\{c_i, E\}}{n}$.

Principles / Rules	$CELmin$	CEL
Oreder preservation	Yes	Yes
Resource monotonicity	Yes	Yes
Super modularity	Yes	Yes
Order preservation under claims variation	Yes	Yes
Composition down	Yes	Yes
Limited consistency	Yes	Yes
Composition up	Not	Yes
Invariance under claims truncation	Not	Yes
Self-duality	Not	Not
Midpoint property	Not	Not
Reasonable lower bounds on awards	Not	Not

Table 1: **Properties and rules.** The table shows the principles satisfied by the rules. The two columns correspond to the CELmin and the CEL rules, and each row shows one of the proposed principles.

5. Distribution of the European Regional Development Founds

The ERDF budget of the European council and Parliament determined for programming period 2014-2020 is approximately 182.150 million euros, which corresponds to almost 44% of the total budget. This budget flows to the second level of the EU common classification of territorial units for statistics (NUTS2) which involves the regions with population between 800,000 and 3,000.000 inhabitants. According to this division, the regional eligibility for the ERDF is calculated taking into account the regional per capita GDP. Regions in NUTS level 2 are split and classified into three different categories according to their *GDP per capita* measured in purchasing power standards, as follows:

More developed regions ($R1$): with GDP per capita above 100 % of the average GDP per capita of the EU-27 .

Transition regions ($R2$): with GDP per capita between 75 % and 100 % of the average GDP per capita of the EU-27 .

Less developed regions ($R3$): with GDP per capita less than 75 % of the average GDP per capita of the EU-27 .

According to this classification, there where 265 regions in NUTS 2 for the programming period 2014-2020. This number declines to 47 if the regions of the same category unify in each country (Solís-Baltodano et al., 2021). From the claims problems perspective, these 47 regions form the claimants who have a claim on the ERDF budget. We use the same claims that Solís-Baltodano et al. (2021) offer in their study. In their method, each agent claims a fixed amount which is equal for all regions, the allovation per inhabitant obtained for the region with the higest GDP per inhabitant (it can be interpreted as a minimal allocation), plus an amount that depends on the gap between the specific region GDP per capita and the highest GDP per capita. The attribute of this method is that the less developed regions claim more than the others. The claims of the regions are depicted in Table 3. Moreover, the table illustrates a comparison between the regional allocation of our proposed rule and the rules that have been already studied. The absolute allocation of ERDF to each country is provided in Table 2.

The CEA rule distributes the fund as equal as possible to all regions without taking into account the measure of their demands. In contrast, the CEL rule imposes equal losses to all regions. Therefore, it helps regions with larger claims, which are regions in $R3$ to obtain more ERDF. But, in this case, the rule causes that some more developed regions ($R1$) receive nothing. Notice that, the rest of the studied rules, P, α_{min} and CELmin stand somewhere between CEA and CEL. In particular, as Table 3 represents, the total ERDF that CELmin allocates to $R3$ regions is equal to the allocation of the CEL with a slight difference. Nonetheless, CELmin supports some regions in $R1$ who are ignored by the CEL (e.g. Czechia $R1$). Our main objective is to propose a new rule to distibute the ERDF budget that solves this situatuation. With the CELmin every region receives the minimal right that CELmin guarantees for all regions.

Country	Current		P		CEA		CEL		$\alpha^{\min}$		$CEL^{\min}$	
Austria	536.26	(0.29%)	2,990.30	(1.64%)	3,605.90	(1.98%)	1,664.43	(0.91%)	3,023.98	(1.66%)	1,656.43	(0.91%)
Belgium	953.01	(0.52%)	4,153.60	(2.28%)	4,658.91	(2.56%)	3,047.36	(1.67%)	4,181.13	(2.30%)	3,037.02	(1.67%)
Bulgaria	3,567.67	(1.96%)	3,714.38	(2.04%)	2,881.54	(1.58%)	5,426.21	(2.98%)	3,668.28	(2.01%)	5,419.82	(2.98%)
Croatia	4,321.50	(2.37)%	2,130.59	(1.17%)	1,678.03	(0.92%)	3,059.63	(1.68%)	2,105.53	(1.16%)	3,055.90	(1.68%)
Cyprus	299.90	(0.16%)	373.82	(0.21%)	353.24	(0.19%)	413.15	(0.23%)	372.66	(0.20%)	412.37	(0.23%)
Czechia	11,940.69	(6.56%)	4,556.74	(2.50%)	4,336.62	(2.38%)	5,352.62	(2.94%)	4,544.31	(2.49%)	5,386.12	(2.96%)
Denmark	206.62	(0.11%)	1,968.98	(1.08%)	2,362.93	(1.30%)	1,119.93	(0.61%)	1,990.53	(1.09%)	1,114.69	(0.61%)
Estonia	1,856.56	(1.02%)	595.11	(0.33%)	539.17	(0.30%)	706.46	(0.39%)	591.99	(0.33%)	705.27	(0.39%)
Finland	486.64	(0.27%)	2,092.38	(1.15%)	2,253.37	(1.24%)	1,731.82	(0.95%)	2,101.10	(1.15%)	1,726.82	(0.95%)
France	7,978.14	(4.38%)	26,643.93	(14.6%)	27,395.44	(15.0%)	24,782.70	(13.6%)	26,683.47	(14.7%)	24,721.92	(13.57%)
Germany	10,773.84	(5.91%)	29,153.27	(16.0%)	33,839.48	(18.6%)	18,992.84	(10.4%)	29,409.23	(16.2%)	18,917.76	(10.39%)
Greece	8,622.33	(4.73%)	5,203.71	(2.86%)	4,390.21	(2.41%)	6,859.19	(3.77%)	5,158.57	(2.83%)	6,849.45	(3.76%)
Hungary	10,756.78	(5.91%)	4,675.35	(2.57%)	3,996.69	(2.19%)	6,052.90	(3.32%)	4,637.67	(2.55%)	6,044.03	(3.32%)
Ireland	410.78	(0.23%)	902.04	(0.50%)	1,974.31	(1.08%)	0.00	(0.00%)	961.01	(0.53%)	156.50	(0.09%)
Italy	21,507.18	(11.8%)	25,193.38	(13.8%)	24,721.44	(13.6%)	25,919.76	(14.2%)	25,165.68	(13.8%)	25,864.92	(14.20%)
Latvia	2,401.25	(1.32%)	933.67	(0.51%)	790.63	(0.43%)	1,224.56	(0.67%)	925.74	(0.51%)	1,222.81	(0.67%)
Lithuania	3,501.41	(1.92%)	1,276.29	(0.70%)	1,148.07	(0.63%)	1,532.40	(0.84%)	1,269.15	(0.70%)	1,529.85	(0.84%)
Luxembourg	19.50	(0.01%)	6.31	(0.00%)	19.50	(0.01%)	0.00	(0.00%)	19.50	(0.01%)	19.50	(0.01%)
Malta	384.35	(0.21%)	195.76	(0.11%)	194.43	(0.11%)	196.49	(0.11%)	195.67	(0.11%)	196.06	(0.11%)
Netherlands	510.28	(0.28%)	5,750.65	(3.16%)	7,022.38	(3.86%)	3,016.17	(1.66%)	5,820.26	(3.20%)	3,000.59	(1.65%)
Poland	40,213.87	(22.1%)	18,186.09	(9.98%)	15,552.10	(8.52%)	23,654.01	(13.0%)	18,038.20	(9.90%)	23,720.33	(13.02%)
Portugal	10,661.23	(5.85%)	4,778.73	(2.62%)	4,206.22	(2.31%)	5,932.01	(3.26%)	4,746.89	(2.61%)	5,922.68	(3.25%)
Romania	10,726.08	(5.89%)	9,586.80	(5.26%)	7,983.86	(4.38%)	12,855.80	(7.06%)	9,497.91	(5.21%)	12,896.23	(7.08%)
Slovakia	7,291.46	(4.00%)	2,569.88	(1.41%)	2,224.75	(1.22%)	3,367.28	(1.85%)	2,550.70	(1.40%)	3,384.02	(1.86%)
Slovenia	1,416.69	(0.78%)	906.04	(0.50%)	844.79	(0.46%)	1,025.26	(0.56%)	902.61	(0.50%)	1,023.38	(0.56%)
Spain	20,079.13	(11.0%)	20,013.10	(11.0%)	19,070.57	(10.5%)	21,783.55	(12.0%)	19,959.85	(11.0%)	21,741.24	(11.94%)
Sweden	727.83	(0.40%)	3,600.08	(1.98%)	4,136.42	(2.27%)	2,432.44	(1.34%)	3,629.36	(1.99%)	2,425.27	(1.33%)

Table 2: Absolute allocations of ERDF funds by country: current allocations and proposals according to the different rules (in M€). Between brackets there is the percentage of the funds allocated to each country.

Country	Region	Claim	Current	P	CEA	CEL	$\alpha^{\min}$	$CEL^{\min}$
Austria	R1	1,038	57.4	335.79	408.73	178.90	339.78	178.00
Austria	R2	1,332	160.6	430.93	408.73	473.05	429.67	472.15
Belgium	R1	1,009	36.3	326.20	408.73	149.25	330.72	148.35
Belgium	R2	1,425	203.1	460.89	408.73	565.70	457.99	564.80
Bulgaria	R3	1,629	506	526.86	408.73	769.67	520.32	768.76
Croatia	R3	1,605	1,052.6	518.96	408.73	745.25	512.86	744.35
Cyprus	R1	1,337	347	432.54	408.73	478.05	431.20	477.15
Czechia	R1	565	244,8	182,70	408,73	0	195,13	32.40
Czechia	R3	1.434	1.247,8	463,77	408,73	574,59	460.70	573.68
Denmark	R1	1,004	33.3	324.65	408.73	144.45	329.26	143.55
Denmark	R2	1,345	50.3	434.97	408.73	485.55	433.49	484.65
Estonia	R2	1,395	1,407.4	451.14	408.73	535.55	448.77	534.65
Finland	R1	1,173	88.2	379.53	408.73	314.13	381.11	313.22
France	R1	1,142	67.6	369.35	408.73	282.68	371.50	281.77
France	R2	1,418	145.3	458.69	408.73	558.90	455.91	558.00
France	R3	1,530	1,003.5	494.80	408.73	670.55	490.03	669.65
Germany	R1	1,039	61.4	335.99	408.73	179.51	339.97	178.61
Germany	R2	1,351	491.4	436.89	408.73	491.50	435.31	490.59
Greece	R1	1,327	419	429.03	408.73	467.20	427.89	466.29
Greece	R2	1,573	795.6	508.58	408.73	713.16	503.05	712.26
Greece	R3	1,622	1,181.9	524.53	408.73	762.48	518.12	761.57
Hungary	R1	1,202	85.6	388.88	408.73	343.05	389.95	342.15
Hungary	R3	1,601	1,551.5	517.85	408.73	741.83	511.81	740.92
Ireland	R1	577	85	186.74	408.73	0	198.95	32.40
Italy	R1	1,158	91.2	374.43	408.73	298.36	376.29	297.46
Italy	R2	1,440	277	465.85	408.73	581.04	462.67	580.13
Italy	R3	1,556	973.9	503.39	408.73	697.12	498.15	696.21
Latvia	R3	1,492	1,241.4	482.67	408.73	633.05	478.57	632.15
Lithuania	R3	1,405	1,246.5	454.37	408.73	545.55	451.83	544.65
Luxembourg	R1	32.4	32.4	10.48	32.40	0	32.40	32.40
Malta	R2	1,272	808	411.52	408.73	413.05	411.34	412.15
Netherlands	R1	1,035	29.7	335.79	408.73	175.55	338.76	174.65
Poland	R1	840	880.5	271.64	408.73	0	279.17	32.40
Poland	R3	1,536	1074.4	496.81	408.73	676.76	491.93	675.86
Portugal	R1	1,276	274.8	412.61	408.73	416.41	412.36	415.50
Portugal	R2	1,370	514.7	443.05	408.73	510.55	441.13	509.65
Portugal	R3	1,513	1,417.4	489.37	408.73	653.77	484.90	652.86

Table 3: Claims, current allocations, and proposals according to the different rules, in € *per capita*.

Country	Region	Claim	Current	P	CEA	CEL	$\alpha^{\min}$	$CEL^{\min}$
Romania	R1	867	268.4	280.54	408.73	8.05	287.57	32.40
Romania	R3	1,604	586.6	518.88	408.73	745	512.78	744.09
Slovakia	R1	707	402.6	228.79	408.73	0	238.68	32.40
Slovakia	R3	1,562	1,466.8	505.18	408.73	702.65	499.84	701.74
Slovenia	R1	1,225	467.4	396.16	408.73	365.55	396.82	364.65
Slovenia	R3	1,472	928.6	476.21	408.73	613.05	472.46	612.15
Spain	R1	1.248	253,6	403,76	408,73	389,04	404,00	388.14
Spain	R2	1,485	743.1	480.1	408.73	625.21	476.17	624.30
Spain	R3	1,512	1,473	489.14	408.73	653.05	484.90	652.15
Sweden	R1	1,100	71.9	355.77	408.73	240.55	358,62	239.65

Table 3: Claims, current allocations, and proposals according to the different rules, in € *per capita*.

6. Convergence

It is noteworthy to re-emphasize that the objective of the European Union through the ERDF is to elevate the growth rate of less developed regions to from the convergence in EU territory. Supporting the less developed regions requires to detect the division rules which distribute ERDF in the way that is more favorable for larger claimants.

Lorenz dominance is an appropriate criterion which explores how the rules treat smaller claimants relative to larger claimants. A Lorenz dominant rule is an equitable rule which is favorable for smaller claimants.

Let $\mathbb{R}^n_+$ be the set of positive n-dimensional vectors $x = (x_1, x_2, \ldots, x_n)$ ordered from small to large, i.e., $0 < x_1 \leq x_2 \leq \ldots \leq x_n$. Let x and y be in $\mathbb{R}^n_+$. We say that x Lorenz dominates y, $x \succ_L y$, if for each $k = 1, 2, \ldots, n-1$: $x_1+x_2+\cdots+x_k \geq y_1+y_2+\ldots+y_k$ and $x_1+x_2+\ldots+x_n = y_1+y_2+\ldots+y_n$. If x Lorenz dominates y and $x \neq y$, then at least one of these $n-1$ inequalities is a strict inequality. Following definition extends Lorenz dominance to claims problems situations.

Definition 7. *Given two solutions φ and ψ it is said that φ Lorenz dominates ψ, $\varphi \succ_L \psi$, if for any claims problem (E, c) the vector $\varphi(E, c)$ Lorenz dominates $\psi(E, c)$. Indeed, it states that $\varphi(E, c)$ is more equitable and more supportive of smaller claims.*

Bosmans and Lauwers (2011) proved that the CEA as the most equitable rule, since this rule Lorenz dominates all other rules. Their comparison shows that the CEL is the most inequitable rule: $CEA \succ_L \alpha_{\min} \succ_L P \succ_L CEL$.

Next result shows the Lorenz relationships between our solution and the main ones.

Proposition 2.

(a) $CEA \succ_L \alpha_{\min} \succ_L CELmin \succ_L CEL$.

(b) There is no Lorenz dominance relation between CELmin and P.

Proof. Part (a) is easily obtained by definition and previous results. By definition, the CELmin rule has an egalitarian part that makes the rule more equitable than the CEL.

Part (b) is directly obtained from the case analysis. If c_1 is unsustainable, CELmin corresponds to CEA. Therefore, the rule Lorenz dominates P. In case c_1 is sustainable, the result of Table 3 shows that P Lorenz dominates CELmin. ■

Another method that allows us to examine whether the rules promote the convergence is to study the effect of the rules' allocations $x \geq 0$ on the regions' GDP per capita GDP^h. It is expected that after allocating the ERDF to regions, their GDP per capita increase to a new GDP per capita $\widehat{GDP^h}$. Solís-Baltodano et al. (2021) introduce *divergence ratio* which is defined on the basis of the difference between regions' *GDP per capita*. The divergence ratio of less developed region α versus more developed region β is as follow:

$$d_{(\alpha,\beta)} = 1 - \frac{GDP^h_\alpha}{GDP^h_\beta}$$

The divergence ratio is always greater than 0 and the amount close to 0 reflects the convergence in EU. Obviously, the CEA rule distributes the budget in the most egalitarian manner possible, maintaining the existing differences before the budget was allocated. On the contrary, the CEL rule provides the less egalitarian distribution of the funds. Therefore, CEL rules may be most appropriate to achieve the convergence goal of the ERDF. However, as aforementioned, this rule may assign no funds to some regions, which provokes its difficult real implementation. Our new rule, the CELmin, ensures a minimum amount to each region, favouring the decreasing of the gap among regions, and reducing the divergence ratio faster than other rules.

7. Final remarks

Although the Constrained Equal Losses is an appropriate solution when the aim is to support larger claims, but zero allocation to some smaller claims is an obstacle to use this rule in a real situation. To adjust this problem and simultaneously to advocate the larger claims, we propose a rule which assures a fixed allocation right to all agents by compromising the Egalitarian rule and Constrained Equal Losses. In conjunction with clarifying the properties of the rule by axiomatic analysis, we implement it to re-allocate the ERDF. The results depict the rule obtains the convergence in EU while it can support the smaller claims.

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